<Totally Unimodular Matrices (TU Matrices)>

**Definition:** A matrix *A* is *totally unimodular (TU)* if every square submatrix of *A* has determinant +1, -1 or 0.

e.g. ,

**Proposition 1.** A matrix *A* is TU iff

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2. The augmented matrix (A I) is TU.

**Proposition 2. (충분조건 for TU)**

1. Each column contains at most two nonzero coefficients ()
2. There exists a partition () of the set M of rows such that  
   each column j containing two nonzero coefficients satisfies .

(and one of them can be empty.)

Suppose that there is a linear system such that where (integral matrix) and Then, we have the following theorem.

**Hoffman-Kruskal’s *Theorem:***

*The polyhedron is integeral (⇔All the vertices of the feasible region are integral) for every iff A is totally unimodular (TU).*

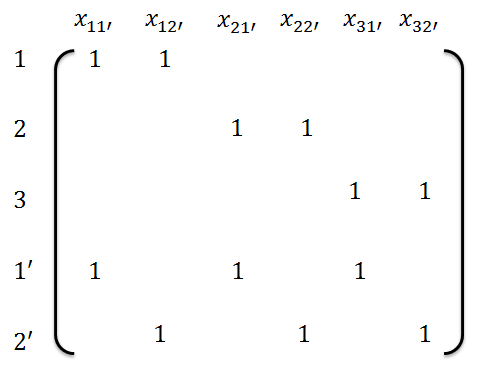
* The proof refers to pp.132 in “**Integer Programming” written by Conforti et. al.**(Check: Cramer’s rule, vertex=basic feasible solution=extreme point, the properties of TU)

**Therefore, if the matrix in a linear system is TU and the vector on the right-handed side in the system is integral, then we always have the optimal solution which is integral only by applying the algorithms for solving LP!**

**It is very important to figure out TU structure in the system which allows us to easily or partially solve the IP problem.**

**Examples>**

1. Transportation Problem: M (|M|=3): suppliers, N: demands (|N|=2)

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1. Network Problem (incidence matrix): Minimum Cost Network Flows

